Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name :Engineering Mathematics - IV

Subject Code :4TE04EMT1 Branch:B.Tech (Auto,Mech,EEE,EE,IC,Civil,EC)

Semester : 4 Date : 07/05/2016 Time : 2:30 To 5:30 Marks : 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Write Fourier sine transform of $f(t)$. **(01)**
b) A vector $\vec{F}$ is solenoidal if _____ **(01)**
c) In usual notation $E = 1 - \nabla$. True or False **(01)**
d) The function $\bar{z}$ is not analytic at any point. True or False? **(01)**
e) The function $e^x \cos y$ is not harmonic. True or False? **(01)**
f) The region $|z| \leq 1$ represent open unit disk. True or False? **(01)**
g) Range – Kutta method is better than Tayler's method. True or False? **(01)**
h) The convergence in the Gauss – Seidal method is faster than Gauss – Jacobi method. True or False? **(01)**
i) If $\phi = 3x^2y - y^3z^2$, find $gradient\ \phi$ at the point (1,-2,1) **(02)**
j) State Green's theorem. **(02)**
k) If $y = 3x^3 - 2x^2 + 1$ find $\Delta^3 y$. **(02)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find the Fourier cosine integral of $f(x) = e^{-kx}\ \ (x > 0,\ k > 0)$. Using that evaluate $\int_0^\infty \frac{\cos\ \lambda x}{k^2 + \lambda^2}\ d\lambda$ **(05)**

b) Solve the one dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}, -\infty < x < \infty,\ t > 0$ with the initial conditions $u(x,0) = f(x), \frac{\partial u\,(x,0)}{\partial t} = g(x)$ and the boundary conditions $u, \frac{\partial u}{\partial x} \to 0$as $x \to \pm\infty$. **(05)**

c) Find the Fourier transform of $f(x)$ if $f(x) = \begin{cases} 0 & 0 < x < a \\ x & a \leq x \leq b \\ 0 & x > b \end{cases}$ **(04)**

(14)

Q-3 **Attempt all questions**

a) Determine analytic function whose imaginary part is $e^x\ (x\cos y - y\sin y)$ **(05)**

b) If $f(z) = u + iv$ is an analytic function of z and $u + v = e^x\ (cos\, y + sin\, y)$, find $f(z)$. **(05)**

c) Find p such that the function $f(z) = r^2\cos 2\theta + i\, r^2 \sin p\theta$ is analytic. **(04)**

Q-4 **Attempt all questions** **(14)**

a) Under the transformation $w = \frac{1}{z}$ **(05)**

i. Find the image of $|z - 2i| = 2$.

ii. Show that the image of the hyperbola $x^2 - y^2 = 1$ is the lemniscate $\rho^2 = \cos 2\theta$

b) Find the bilinear transformation which sends the points $z = 0, 1, \infty$ in to the points $w = -5, -1, 3$ respectively. What are the invariant points of the transformation? **(05)**

c) Following table gives the values of x and y: **(04)**

x	1.0	1.05	1.10	1.15	1.20	1.25	1.30
y	1.00	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

Find $\frac{dy}{dx}$ for $x = 1.05$ using forward difference.

Q-5 **Attempt all questions** **(14)**

a) Solve by Gauss – Jordan method **(05)**

$5x - 2y + 3z = 18, \quad x + 7y - 3z = -22, \quad 2x - y + 6z = 22.$

b) Solve the equation **(05)**

$27\,x + 6y - z = 85, 6x + 5y + 2z = 72, x + y + 54z = 110$ by Gauss – Seidel method.

c) If $\vec{F} = (x + y + 1)i + j - (x + y)k$ find $\vec{F} \cdot curl\ \vec{F}$. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Verify Green's theorem for the function $\vec{F} = (x + y)\ i + 2xy\ j$ and C is the rectangle in XY −plane bounded by $x = 0, y = 0, x = a, y = b$. **(07)**

b) Verify Stokes's theorem for $\vec{A} = (2x - y)i - yz^2\ j - y^2 z\ k$, where S is the upper half surface of sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **(07)**

Q-7 **Attempt all questions** **(14)**

a) Use the fourth – order RungeKutta method to solve $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1.$ **(05)**

Evaluate the value of y when $x = 0.1$

b) Find the value of y for $x = 0.1$ by Picard's method, given that **(05)**

$$\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$$

c) Following table gives the values of x and y: **(04)**

x	30	35	40	45	50
y	15.9	14.9	14.1	13.3	12.5

Find value of x corresponding to $y = 13.6$

Q-8 **Attempt all questions** **(14)**

a) Construct Newton's forward interpolation polynomial for the following data: **(05)**

X	4	6	8	10
Y	1	3	8	16

Use it to find the value of y for $x = 5$.

b) Use Lagrange's interpolation formula to find the value of y when $x = 10$, if the values of x and y are given below: **(05)**

x	5	6	9	11
y	12	13	14	16

c) Divide the range into 10 equal parts, find the approximate value of $\int_0^{\pi} \sin x \ dx$ by Simpson's $\frac{1}{3}$ rule. **(04)**